

# GOCESeaComb

External calibration/validation of ESA's GOCE mission and contribution to DOT and SLA determination through stochastic combination with heterogeneous data



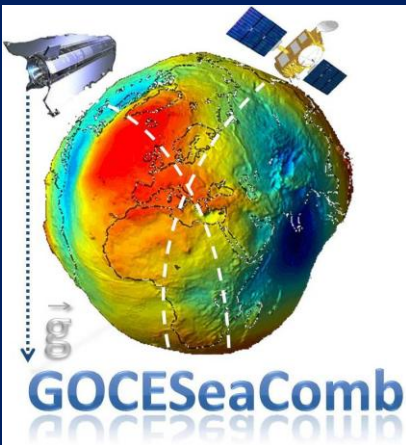
Newsletter Issue 10/28.02.2014

## GOCESeaComb

The **GOCESeaComb** project is funded by the European Space Agency (ESA) within its Scientific Experiment Development Program (PRODEX) following a successful application to the General Secretariat for Research & Technology (GSRT) after an invitation to the Greek scientific community in response to the 1st PRODEX Programme Call for Greece.

Contract: C4000106380

Duration: July 2012 – July 2014



The GOCESeaComb Project Logo

## MDOT AND TDOT COVARINACE FUNCTION DETERMINATION

During the period of this newsletter and since the last newsletter in December 2013, all project activities are going according to schedule. Having completed the detailed validation of the GOCE/GRACE GGMs and the determination of the empirical and analytical covariance functions for the MDOT, in this period the analysis of existing TDOT models has been carried out. This refers to the determination of empirical and analytical covariance functions so that the TDOT information will be incorporated within an LSC adjustment scheme.

## TDOT ANALYTICAL COVARINACE FUNCTION MODELS

In order to determine some analytical model for the TDOT covariance function, various options have been tested. In all cases the analytical covariance function models should agree to the empirical values available for the area under study in order to represent the local statistical characteristics of the  $\Delta\zeta$ . The first class of analytical models, refers to exponential ones and the second one refers to 2<sup>nd</sup> and 3<sup>rd</sup> order Gauss-Markov. For the exponential ones six choices were examined, with varying number of parameters to be determined as follows:

$$C_{\Delta\zeta\Delta\zeta}(t) = ae^{bt}$$

$$C_{\Delta\zeta\Delta\zeta}(t) = ae^{bt} + ce^{dt}$$

$$C_{\Delta\zeta\Delta\zeta}(t) = ae^{\left(\frac{t-b}{c}\right)^2}$$

$$C_{\Delta\zeta\Delta\zeta}(t) = ae^{-bt^2}$$

$$C_{\Delta\zeta\Delta\zeta}(t) = ae^{-bt} \cos(\omega t)$$

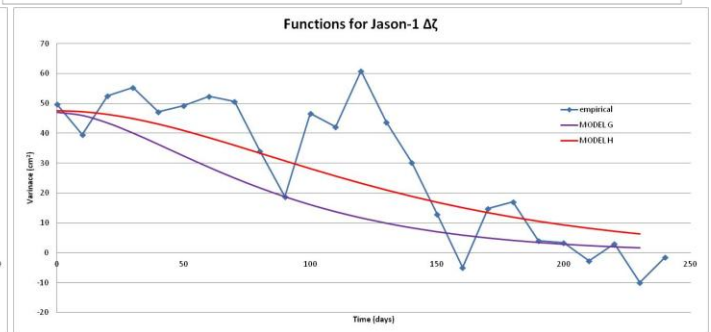
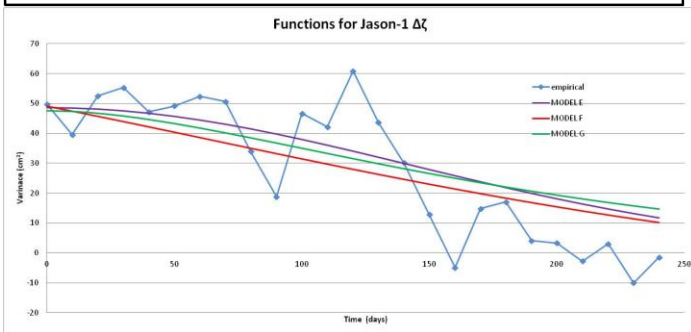
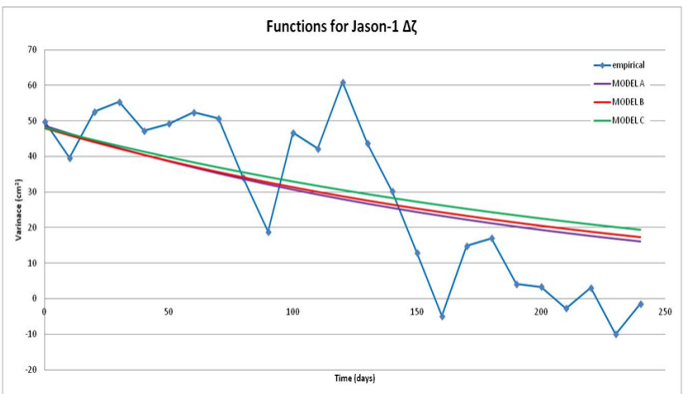
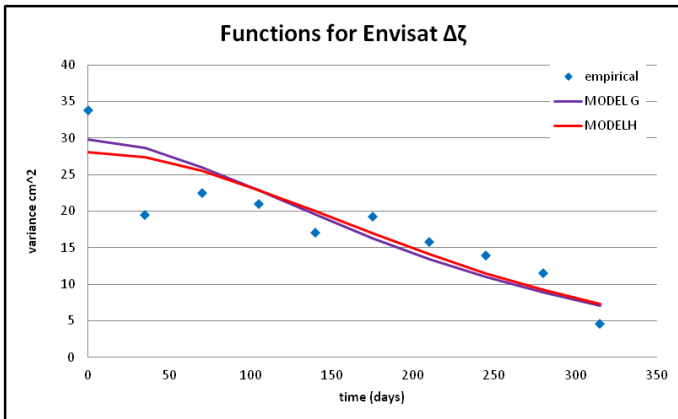
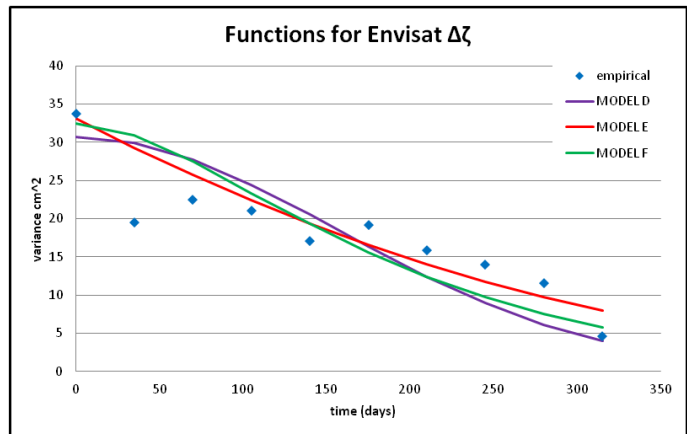
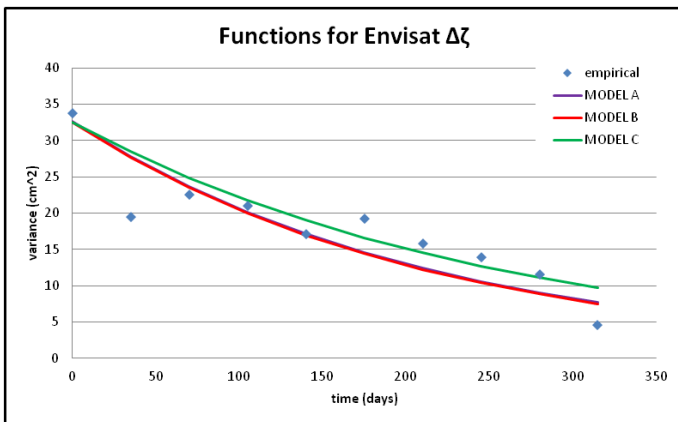
$$C_{\Delta\zeta\Delta\zeta}(t) = \alpha(1 + bt)e^{-bt}$$

In the above equations,  $a$ ,  $b$ ,  $c$  and  $d$  denote parameters to be determined so that the analytical covariance model will fit the empirical one. Note that all above models are a function of the difference in the time between each observation of TDOT. The 2<sup>nd</sup> and 3<sup>rd</sup> order Gauss-Markov models are outlined below, where  $T$  is the characteristic time,  $t$  is the relative time and  $\sigma_{\Delta\zeta}^2$  the  $\Delta\zeta$  variance.

$$C_{\Delta\zeta\Delta\zeta}(t) = \sigma_{\Delta\zeta}^2 \left( 1 + \frac{t}{T} \right) e^{(-t/T)}$$

$$C_{\Delta\zeta\Delta\zeta}(t) = \sigma_{\Delta\zeta}^2 \left( 1 + \frac{t}{T} + \frac{t^2}{3T^2} \right) e^{(-t/T)}$$

All aforementioned models have been evaluated for the  $\Delta\zeta$ . The results presented below refer consecutively to each model in order to demonstrate the performance of the analytical models. After having determined the empirical function, the analytical models are fitted to the empirical values. The six exponential models are denoted as MODEL A, B, ..., F respectively in the sequel and the 2<sup>nd</sup> and 3<sup>rd</sup> order Gauss-Markov models are denoted as MODEL G, H.



In the Figures above, the empirical covariance functions of the  $\Delta\zeta$  are depicted with blue dots for Envisat model and with a blue line with dots for Jason-1 model, along with the fitted analytical models (Models A, B, ...H). For Envisat model, all models provide good fit to the empirical values. This is logical as the form of empirical covariance function is close to this of an exponential model and as a result all models fit well to the empirical one. On the other hand, for Jason-1, the miss-fitting of the models in the biggest part of the equation is obvious due to the badly scaled covariance function.

After the determination of empirical covariance functions and the fitting of analytical models to the empirical values, prediction is carried out with LSC so as to get some results about the accuracy of each model. Prediction has been carried out by omitting every second point where values of  $\Delta\zeta$  are available using the rest for the prediction. Prediction has been made for two models of  $\Delta\zeta$  for all available data. Due to the badly scaled and close to singular matrices, prediction for the model of Jason-1 with all models has been made using the method of singular value decomposition for the inversion of matrices. This method has also been applied to model D and model F for Envisat model due to problems in the inversion of the matrix (ill-posed matrices). In the Tables below statistics of the models and the prediction errors from the various analytical models for all test cases are presented. All models present small prediction errors. This fact can be attributed to the existence of many data in the region where predictions need to be made and to the small differences on the time of nearly observations. Difference on time between two continuous observations is of the order of  $10^{-5}$  days and as a result predicted values are close to the input data. On the other hand, the small errors with std of the order of a few mm indicate that the determined analytical covariance functions perform well, so that the estimates determined are rigorous and robust.

Statistics of model of Envisat ( $\Delta\zeta$ ) and prediction errors from the various analytical models. Unit: [cm].

<b>Envisat <math>\Delta\zeta</math></b>				
	<b>min</b>	<b>max</b>	<b>mean</b>	<b>std</b>
<b><math>\Delta\zeta</math></b>	<b>-27.1</b>	<b>26.4</b>	<b>3.4</b>	<b><math>\pm 10.3</math></b>
<b>MODEL A</b>	-8.200	10.497	0.165	$\pm 2.299$
<b>MODEL B</b>	-8.200	10.497	0.165	$\pm 2.299$
<b>MODEL C</b>	-8.200	10.497	0.165	$\pm 2.299$
<b>MODEL D</b>	-19.971	19.694	0.129	$\pm 6.329$
<b>MODEL E</b>	-8.200	10.497	0.165	$\pm 2.299$
<b>MODEL F</b>	-22.018	14.438	0.139	$\pm 5.521$
<b>MODEL G</b>	-8.200	10.497	0.165	$\pm 2.299$
<b>MODEL H</b>	-8.200	10.497	0.165	$\pm 2.299$

Statistics of model of Jason-1 ( $\Delta\zeta$ ) and prediction errors from the various analytical models. Unit: [cm].

<b>Jason-1 <math>\Delta\zeta</math></b>				
	<b>min</b>	<b>max</b>	<b>mean</b>	<b>std</b>
<b><math>\Delta\zeta</math></b>	<b>-36.024</b>	<b>32.367</b>	<b>5.325</b>	<b><math>\pm 11.161</math></b>
<b>MODEL A</b>	-19.032	11.051	0.010	$\pm 2.033$
<b>MODEL B</b>	-19.032	11.051	0.010	$\pm 2.033$
<b>MODEL C</b>	-19.032	11.051	0.010	$\pm 2.033$
<b>MODEL D</b>	-30.693	27.789	0.027	$\pm 9.025$
<b>MODEL E</b>	-19.032	11.051	0.010	$\pm 2.033$
<b>MODEL F</b>	-16.479	25.407	0.015	$\pm 4.606$
<b>MODEL G</b>	-19.032	11.051	0.010	$\pm 2.033$
<b>MODEL H</b>	-16.479	25.407	0.015	$\pm 4.606$

## ACRONYMS

LSC	Least Squares Collocation
MDOT	Mean Dynamic Ocean Topography
TDOT	Time-varying Dynamic Ocean Topography
SLA	Sea Level Anomalies

---

## Contact Us

### GeoGrav - AUTH

Department of Geodesy and Surveying, Aristotle University of Thessaloniki

University Campus, University Box 440, GR-54124

Thessaloniki, Greece

T: ++302310996125 | F: ++302310995948

[tziavos@topo.auth.gr](mailto:tziavos@topo.auth.gr) ✉ [vergos@topo.auth.gr](mailto:vergos@topo.auth.gr)

<http://olimpia.topo.auth.gr/GOCESeaComb/>