Sea level anomaly and dynamic ocean topography analytical covariance functions in the Mediterranean Sea from ENVISAT data

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Introduction and Problem

The determination, monitoring and understanding of sea level change at various spatial and temporal scales have been the focus of many studies during the past decades.

The advent of satellite altimetry and the multitude of unprecedented in accuracy and resolution observations that it offers allowed precise determinations of sea level variations.

The realization of the GRACE mission and the development of global geopotential models form GOCE observables offer new opportunities for the estimation of sea level trends at regional and global scales and the identification of seasonal signals.

In such studies, a point that has been given little attention is error propagation through analytical data variance-covariance matrices. The latter is of significant importance in heterogeneous data combination studies since error propagation can provide reliable estimates of the output signal error.

This is especially evident in the optimal estimator used in physical geodesy, i.e., least-squares collocation (LSC), where the full variance-covariance matrices are needed for the input data and signals to be predicted.

Given that no analytical models are available for altimetric sea level anomalies (SLA) their incorporation in LSC-based combination schemes is problematic.

Objectives

This work presents some new ideas and results on the determination of analytical covariance functions and subsequently full variance-covariance matrices for the SLAs in the Mediterranean Sea.

The focus is based on single-mission altimetry data from ENVISAT for the entire duration of the satellite mission (2002-2011), both in the along-track direction (see Figure 1 top) and in 2D cases (see Figure 1 bottom).

For the latter, the signal and error characteristics of the sea level anomalies have been studied at monthly, seasonal and annual scales.

The estimation of the analytical covariance functions is performed using 2nd and 3rd order Markov models as well as a kernel similar to that of the disturbing potential a.k.a. dependent on a series of Legendre polynomials.

The same analysis has been carried out for the RioMed dynamic ocean topography (DOT) model available for the entire Mediterranean.

The goal is to come to some conclusions on the SLA and DOT spectral characteristics based on empirically derived properties such as the variance and correlation length and determine analytical models to be used later for prediction with LSC.

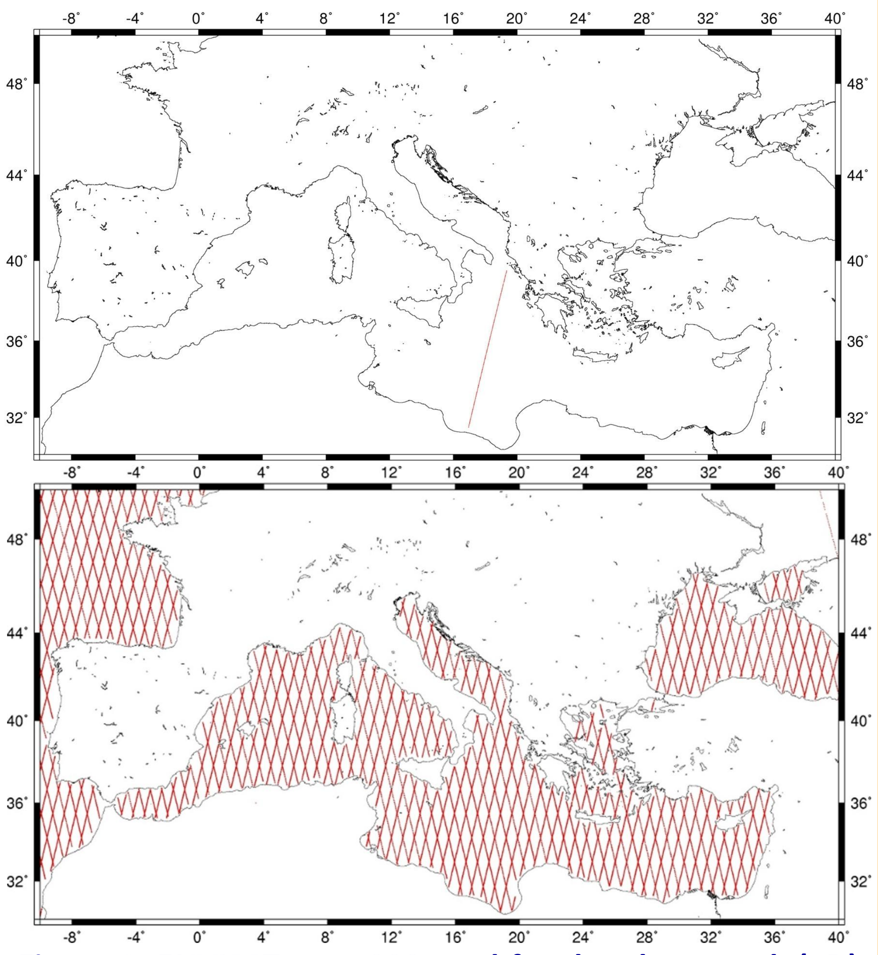

Figure 1: ENVISAT pass 444 used for the along-track (1D) SLA covariance function study (top) and distribution of ENVISAT passes in the Mediterranean Sea (2D case).

Data used and corrections

ENVISAT pass 444 was selected for the along-track study, being the longest one available in the Mediterranean Sea.

The pass consists of ~120-130 observations for each cycle, with few gaps exist in the records, while the study covers the period between 2002 and 2010.

For the 2D case, the entire Mediterranean basin was selected, (30º ≤ φ ≤ 50º and -10º ≤ λ ≤ 40º) for the same period. The total record consists of ~690k observations.

The data have been downloaded from the RADS server (DEOS Radar Altimetry Data System) in the form of SLAs relative to EGM2008, after applying all the necessary geophysical and instrumental corrections.

The global and local IB correction has been applied at a latter pre-processing step.

Empirical covariance functions

For ENVISAT pass 444, empirical covariance functions have been estimated for each satellite cycle for the period 2002-2010. Given the satellite repeat period of 35 days, each year consists of ~11 cycles, which should allow to study the variation of the SLA changes at seasonal and annual scales. An example of the estimated empirical covariance functions is shown below for 2005 and 2006.

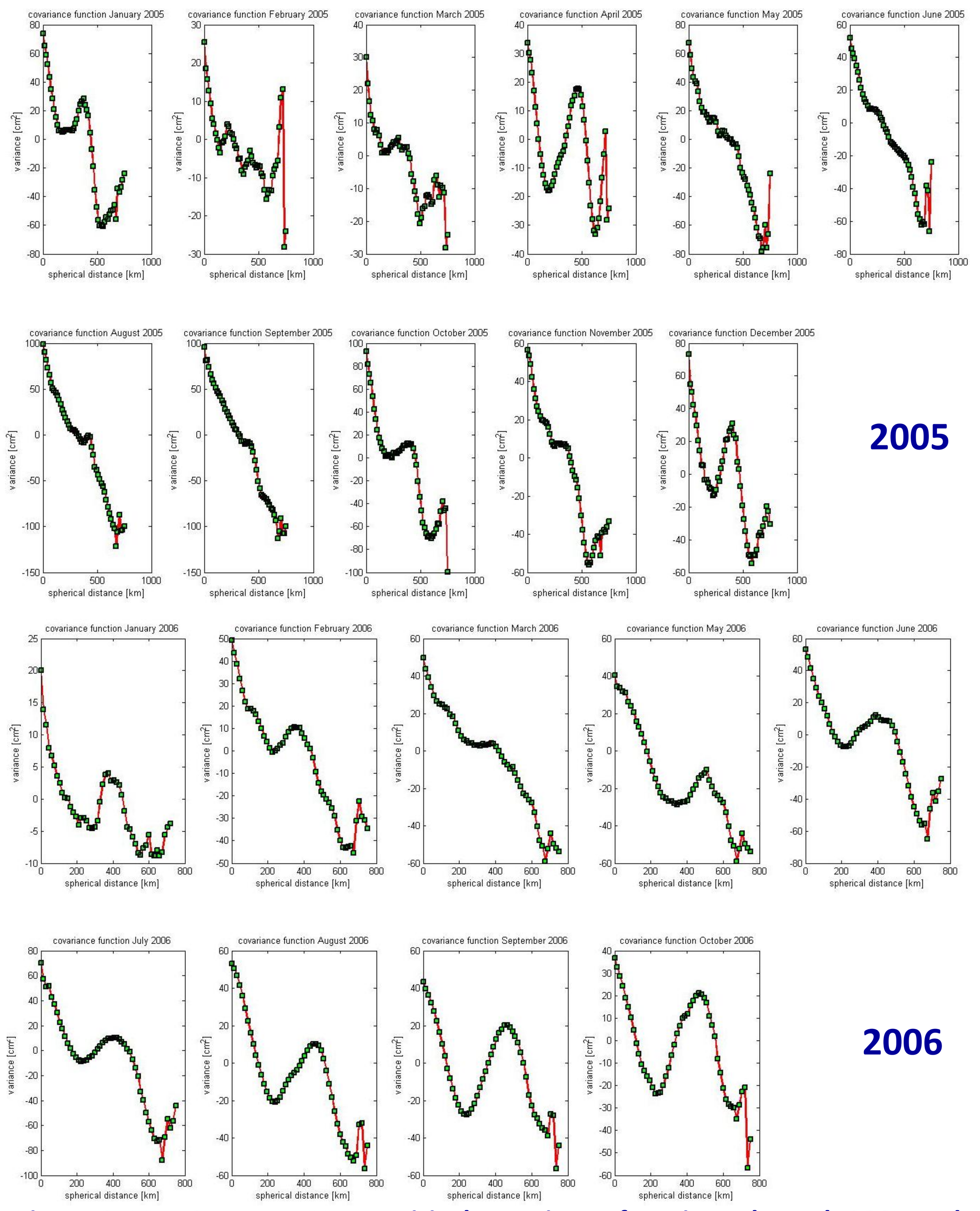

Figure 2: ENVISAT pass 444 empirical covariance functions through 2005 and 2006.

It is interesting to notice how the SLA variance varies through the epochs of its year, with high values in January, lower values in Spring due to reduced rainfall, increasing values as summer progress due to snow melt and the thermal expansion in July-August. Finally, the variance values decrease again in Fall and start increasing in November due to higher level of precipitation.

This evolution through time is shown in the Figure below, where the variances between 2002 and 2010 are presented. It is clear that cyclo-stationarity can be evidenced, showing the repeated behavior of the SLA variations with epochs. The abnormal behavior is correlated with ENSO events (e.g., overall high in August 2008)

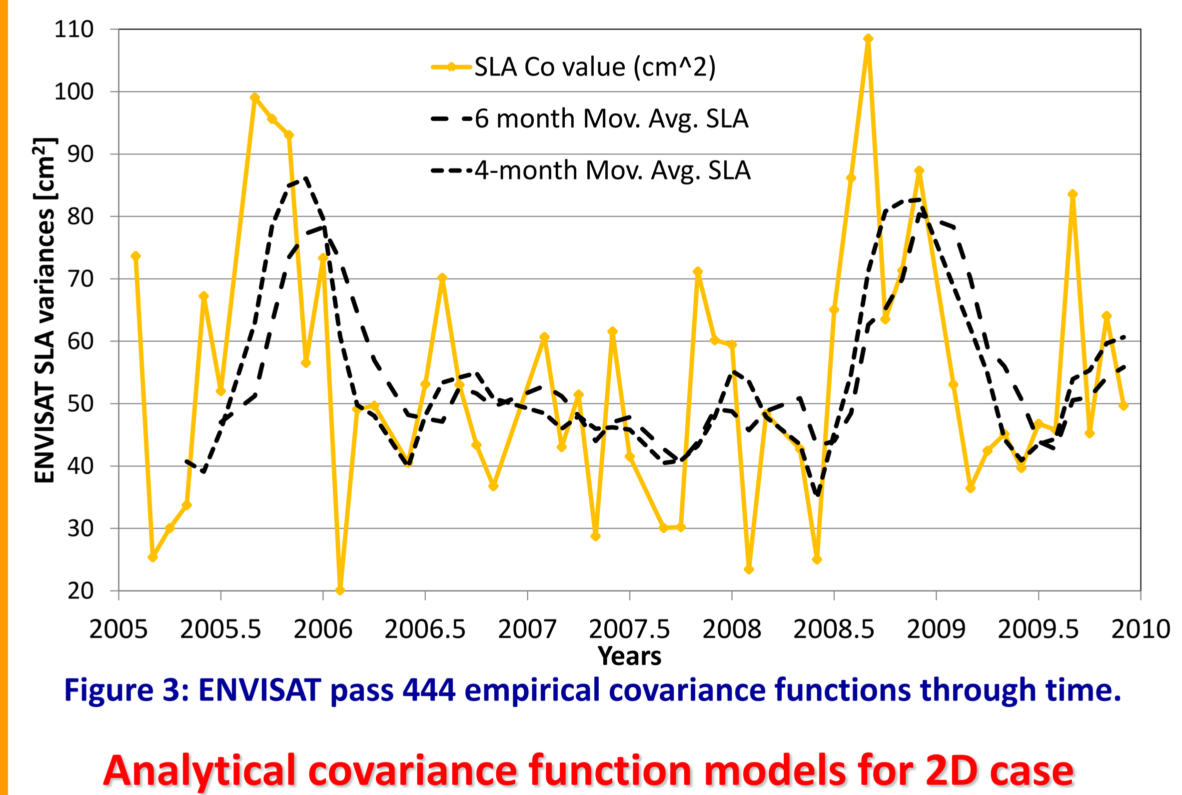

Figure 3: ENVISAT pass 444 empirical covariance functions through time.

Analytical covariance function models for 2D case

As far as the 2D case is concerned, two tests have been carried out. One using a complete cycle of the ENVISAT data for the entire Mediterranean Sea (all passes included, see Figure 1 bottom). This consisted of a total number of 11870 SLA observations, for which analytical covariance functions were determined and predictions were made by omitting every second point and using the rest to estimate the SLA in these locations (TEST D in the sequel).

The second test refers to using the entire set of ENVISAT data, to predict SLA at an inner window where no observations are available. The inner window was selected for the area bounded between (32º ≤ φ ≤ 36º and 15º ≤ λ ≤ 20º). This resembles the case when no information is available in a specific area and LSC is used for the prediction. The validation is performed through comparisons with the available observations (TEST E in the sequel).

Mathematical models and covariance function estimation

First the empirical covariance functions have been determined for the period under study. For the 1D case, along track 444, the empirical covariance function has been estimated and the variance *Co* and correlation length *ξ* for each 35-day pass was determined. The aim is to investigate whether a cyclo-stationarity exists in the SLA data along the same pass for the period 2002-2010.

Then, various analytical covariance function models have been investigated in order to determine the one that provides the overall best fit to the empirical model as well as the optimal results, in terms of prediction accuracy. To this extend, various order exponential models have been studied, along with second and third order Gauss-Markov ones. Apart from planar models, a spherical one based on Legendre polynomial expansion, simulating the Tscherning & Rapp model used to model the analytical covariance function of the disturbing potential was used.

Exponential Models

$C(\psi)=\alpha e^{b\psi}$ **Model A**

$C(\psi)=\alpha e^{b\psi}+ce^{d\psi}$ **Model B**

$C(\psi)=\alpha e^{-\left(\frac{\psi-b}{c}\right)^2}$ **Model C**

$C(\psi)=\alpha e^{-b\psi^2}$ **Model D**

$C(\psi)=\alpha e^{-b\psi}\cos(\omega\psi)$ **Model E**

$C(\psi)=\alpha(1+b\psi)e^{-b\psi}$ **Model F**

Gauss-Markov Models

$$C(r)=\sigma(\bullet)^2\left(1+\frac{r}{D}\right)e^{\left(\frac{-r}{D}\right)}$$ **Model G**

$$C(r)=\sigma(\bullet)^2\left(1+\frac{r}{D}+\frac{r^2}{3D^2}\right)e^{\left(\frac{-r}{D}\right)}$$ **Model H**

Spherical model based on Legendre polynomial expansion

$$C_{(\cdot,\cdot)}(\psi)=\sum_{n=0}^{\infty}\sigma(\bullet)_n^2P_n(\cos\psi) \qquad \sigma(\bullet)_n^2=b\left(\frac{k_2^3}{k_2^3+n_3}+\frac{k_1^3}{k_1^3+n^3}\right)s^{n+1}$$ **Model I**

In the above models, *ψ* denotes the spherical distance, *ξ* the correlation length, *r* the planar distance, *D* the characteristic distance and *σ(•)²* the variance of quantity *(•)* which is under investigation (SLA or DOT). The rest, are parameters to be determined, so that the analytical model will fit the empirical one. Note, that for all models a mixed equations adjustment scheme was used, in order to determine the necessary parameters for each model, based on the empirical values.

Analytical covariance function models for pass 444

An example of the analysis carried out is given in the sequel for pass 444 in August 2005. Figure 4 depicts the SLA as derived from pass 444, where a variation between -30 cm and +30 cm can be seen. For that pass, analytical covariance functions from all aforementioned models were derived and predictions using LSC have been carried our. Three tests have been performed. One, by omitting the first 20 points in the track and using the rest to estimate the SLA in these locations (TEST A in the sequel). The second, by omitting the last 20 points and using the rest to estimate the SLA in these locations (TEST B in the sequel). The third, by omitting every second point and using the rest to estimate the SLA in these locations (TEST C in the sequel).

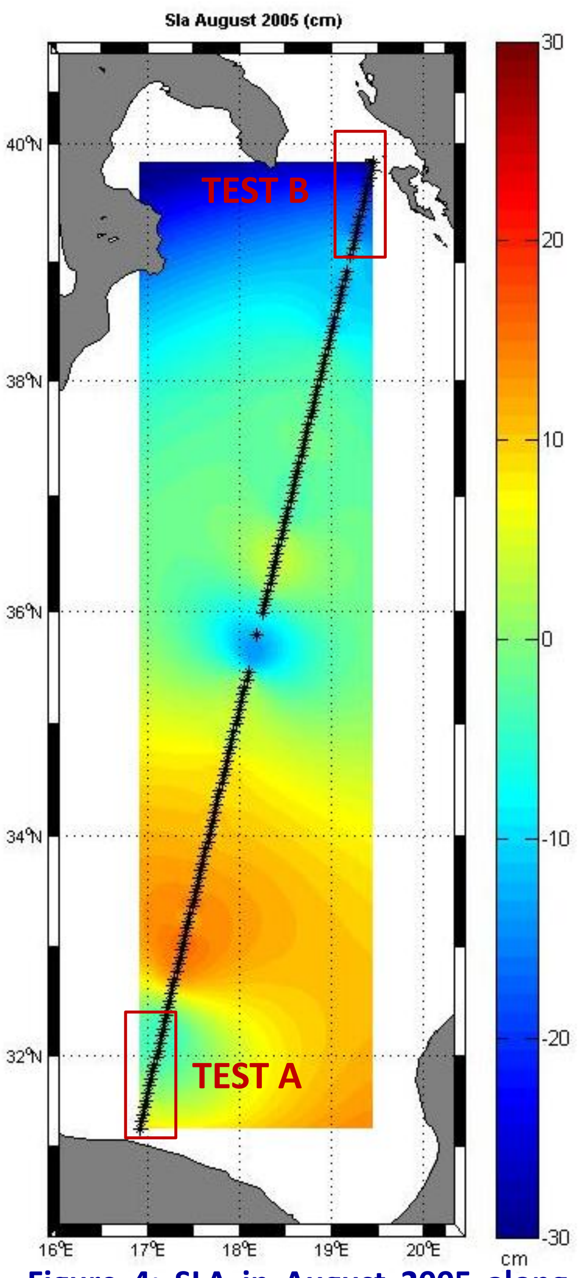

Figure 4: SLA in August 2005 along pass 444.

Statistics of ENVISAT pass 444 SLAs for August 2005 [cm]

	min	max	mean	std
SLA	-19.9	23.5	7.4	±8.5

TEST A

Prediction errors with LSC for the various covariance function models [cm]

	min	max	mean	std
MODEL A	-29.07	3.74	-10.06	±8.87
MODEL B	-18.60	5.07	-4.02	±6.10
MODEL C	-27.75	4.46	-8.53	±8.75
MODEL E	-27.66	4.53	-8.63	±8.67
MODEL F	-22.22	9.53	-0.84	±9.39
MODEL G	-18.97	11.47	1.69	±8.70
MODEL H	-20.76	22.48	7.27	±12.88
MODEL I	-91.3	-2.70	-35.65	±32.5

TEST B

Prediction errors with LSC for the various covariance function models [cm]

	min	max	mean	std
MODEL A	-13.39	5.95	-6.13	±5.54
MODEL B	-128.22	-10.89	-78.97	±35.41
MODEL C	-13.55	6.57	-5.99	±5.78
MODEL E	-13.74	6.38	-6.23	±5.78
MODEL F	-10.57	8.86	-3.15	±5.54
MODEL G	-10.19	10.42	-2.24	±5.94
MODEL H	-15.40	5.79	-7.73	±6.31
MODEL I	22.36	79.54	30.72	±28.59

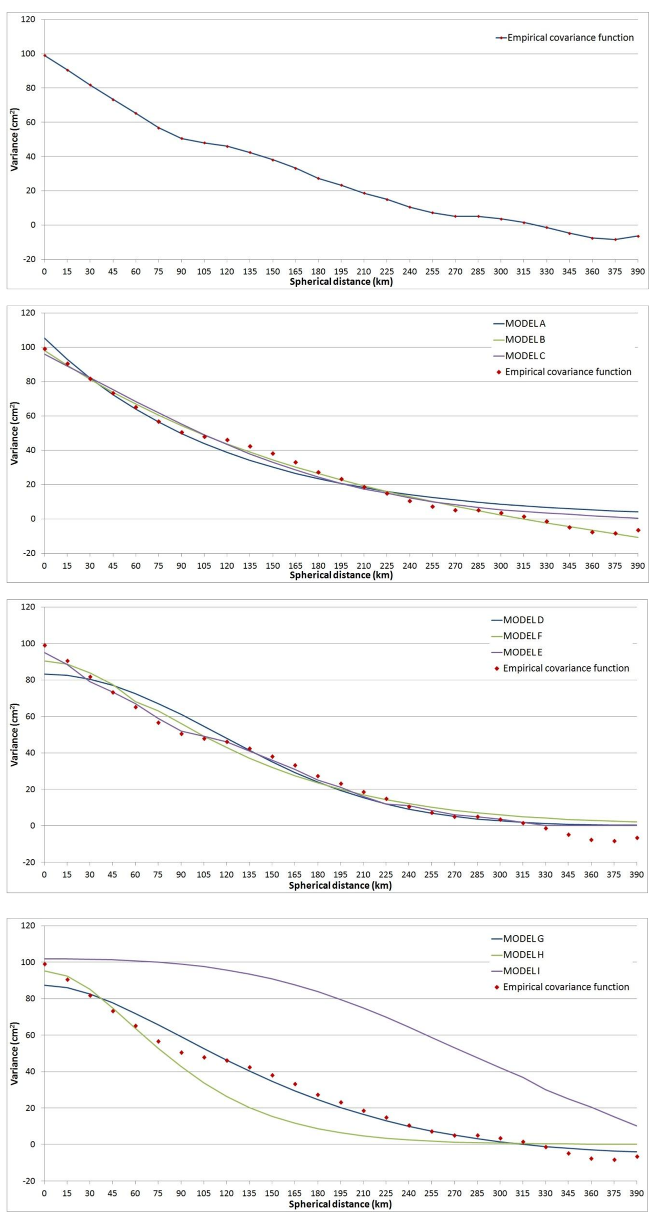

Figure 5: ENVISAT pass 444 empirical and analytical covariance functions for August 2005.

TEST C

Prediction errors with LSC for the various covariance function models [cm]

	min	max	mean	std
MODEL A	-7.61	5.08	-0.11	±1.99
MODEL B	-7.57	5.10	-0.06	±1.88
MODEL C	-7.52	5.09	-0.10	±1.95
MODEL D	-86.59	17.39	-1.43	±12.47
MODEL E	-7.55	5.09	-0.10	±1.95
MODEL F	-8.98	5.27	-0.08	±2.07
MODEL G	-9.00	5.27	-0.07	±2.07
MODEL H	-9.94	5.29	-0.08	±2.18
MODEL I	-11.58	10.37	-0.08	±4.57

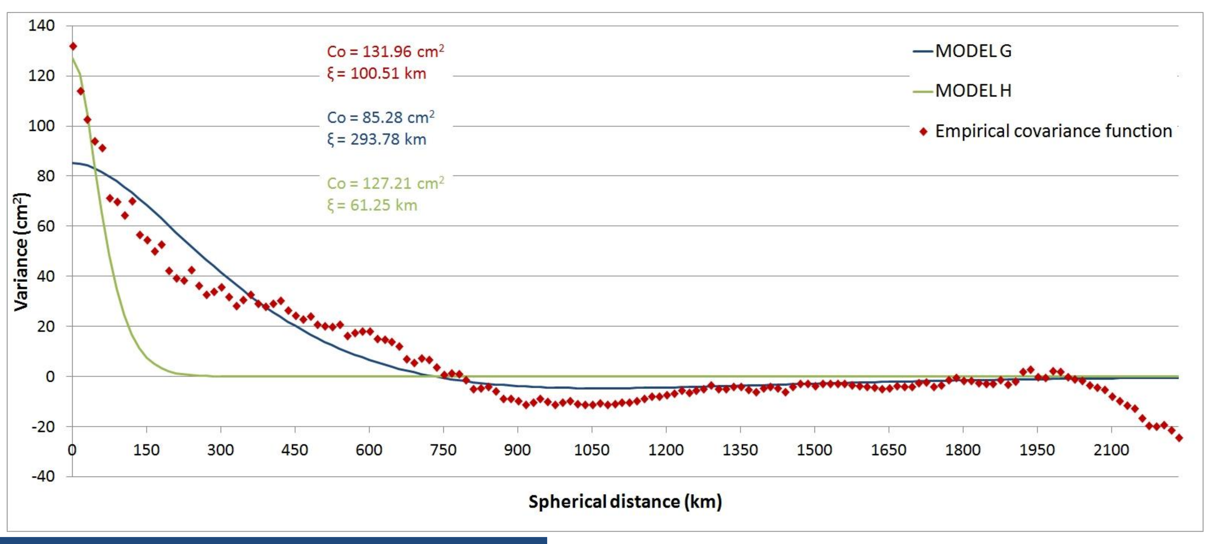

Figure 6: ENVISAT empirical and analytical covariance functions for TEST D.

TEST D

Prediction errors with LSC for the various covariance function models [cm]

	min	max	mean	std
MODEL A	-34.88	29.31	-0.03	±3.65
MODEL B	-34.88	29.37	-0.03	±3.65
MODEL E	-34.89	29.39	-0.03	±3.65
MODEL F	-47.95	55.15	-0.02	±4.57
MODEL G	-47.97	55.18	-0.02	±4.57
MODEL H	-80.37	89.49	-0.02	±5.77

TEST E

Prediction errors with LSC for the various covariance function models [cm]

	min	max	mean	std
MODEL A	-30.91	29.50	0.19	±7.40
MODEL E	-30.91	29.50	0.19	±7.39
MODEL G	-403.55	234.2	1.03	±34.63

Conclusions

- Cyclo-stationarity in the SLA can be evidenced from the empirical covariance functions. The statistical characteristics of the SLA follow a regular annual pattern with the variation of the epochs. Extremes from that is due to the ocean response to ENSO events and atmospheric forcing.
- In the along-track case, the prediction using the exponential analytical covariance function models provide the overall best results, with MODEL E giving the smallest prediction errors.
- The Gauss-Markov models give comparable results in the along-track case and in the 2D case during TEST D, but have one order of magnitude larger errors during TEST E.
- In all cases, the Legendre polynomial expansion for the covariance function given disappointing results since the analytical model does not manage to resemble the pattern of the empirical one.

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