



Estimation of the geopotential value W_0 for the local vertical datum of continental Greece using EGM08 and GPS/leveling data

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Session 5: Establishment and Unification of Vertical Reference Systems

International Symposium on Gravity, Geoid and Height Systems

(GGHS2012), 9-12 October 2012, Venice, Italy



- ***OBJECTIVES***
- ***METHODOLOGY***
- ***THE HELLENIC VERTICAL DATUM***
- ***AVAILABLE DATA***
- ***NUMERICAL RESULTS***
- ***CONCLUSIONS***



Estimate the geopotential value **Wo** for the local vertical datum of continental **Greece**

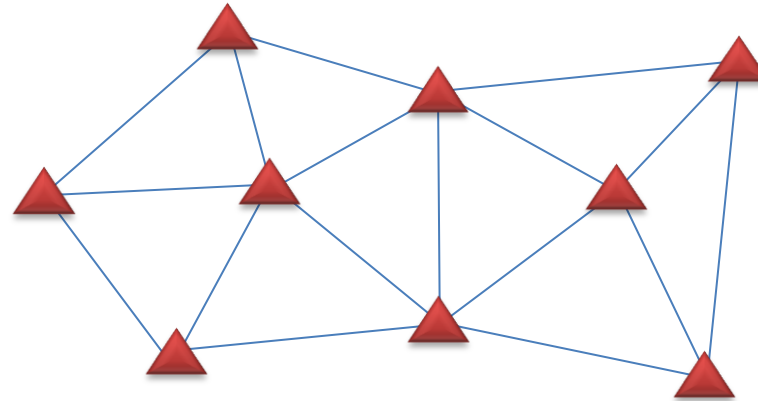
using a **methodology** based on **Helmert heights** and **geopotential models**

in order to allow the **connection** to other local, national, continental and global **height systems**.



Problem setup

Terrestrial
Network



Benchmarks
 $i = \{ 1, 2, \dots, j \}$

Local Vertical datum
 $W=W_0$

H_i (orthometric heights - leveling)

ϕ_i, λ_i, h_i (position – GNSS methods)

g_i, W_i (geopotential model)



W_0 (geopotential value)



By modeling physical heights as Helmert-type orthometric heights:

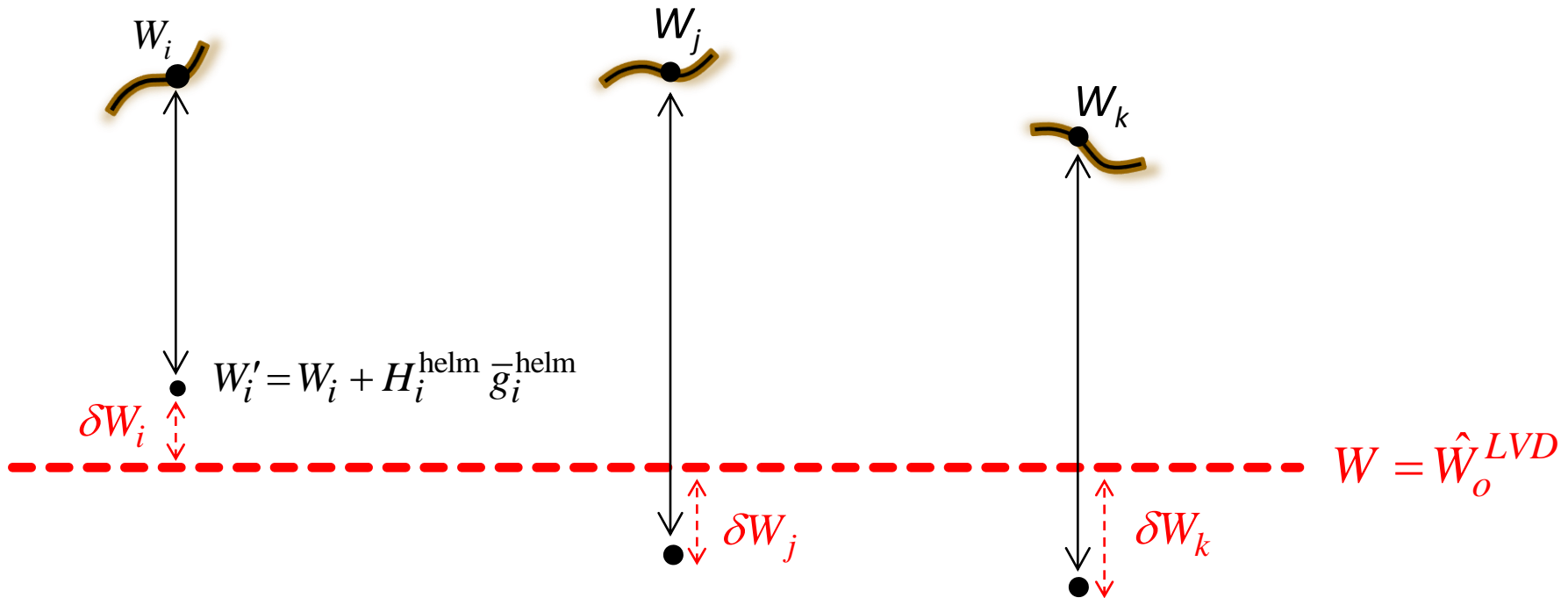
$$H_i^{helm} = \frac{W_o^{LVD} - W_i}{g_i + 0.0424 \cdot 10^{-5} H_i^{helm}}$$

LS estimate of LVD's zero-height level

$$\hat{W}_o^{LVD} = \frac{\sum_i p_i (y_i) \longrightarrow W_i + H_i^{helm} \bar{g}_i^{helm}}{\sum_i (p_i) \longrightarrow \text{'weights'}}$$



Method's rationale



Estimate the LVD zero-height level such that: $\sum_i p_i \delta W_i^2 = \min$



Advantages

- Does not rely on the use of geoid heights and thus is not affected by 'geoid modeling errors'.
- Insensitive to the uncertainty of surface gravity values (g_i), i.e. the accuracy of g_i is not critical and has a small affect to the computed W_o .

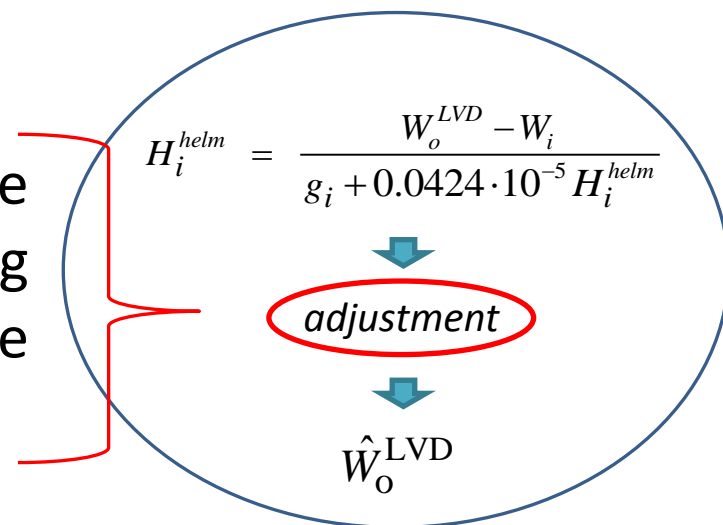
$$dH_i = - \frac{H_i}{g_i + 0.0424 \cdot 10^{-5} H_i} dg_i$$

Even an extreme gravity error of 20mGal at mountainous areas (e.g. $H=2000m$) will affect the height value by 4cm.



Advantages

➔ The residuals obtained after the adjustment can be utilized for studying the error characteristics associated to the physical heights



Additional Notes

- Assumption: Physical heights conform to Helmert-type orthometric heights
- Results obtained from the above methodology may also be tested with corresponding results from other alternative techniques that may employ different geopotential models

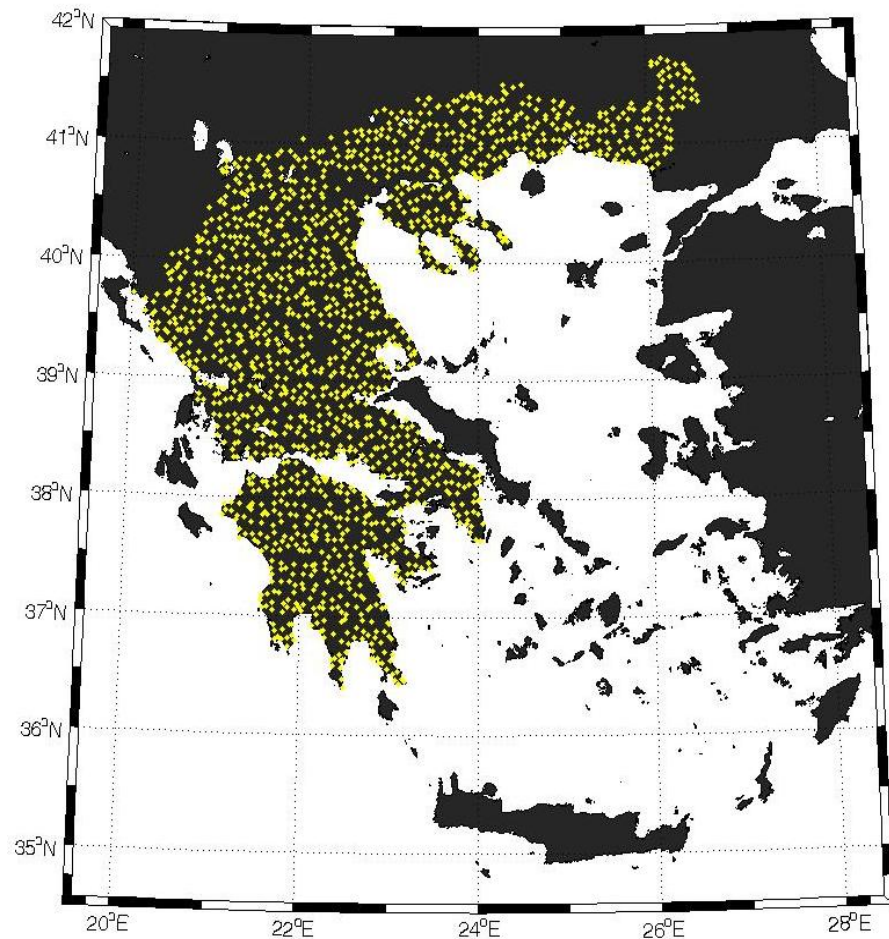


The Hellenic VD was established by the Hellenic Military Service

- ▲ **No scientific documentation** is publically **available**
- ▲ **Unknown W_0** value associated with the Hellenic VD
- ▲ The **accuracy** of **H** is **not** realistically **known**
- ▲ **Height** values are **considered** to **refer** to the **tide-gauge station** located in the **Piraeus** port (Athens) – measurements period 1933-1978.



GPS/Leveling BMs



- 1542 points (*mainland*)
- X, Y, Z (ITRF00, $t=2007.236$) from HEPOS/GPS campaign
- Helmert-type ortho heights from Hellenic Military Service
- Stations with identified blunders/outliers have been removed
- GPS Accuracy (1σ): 1-4 cm (horiz)
2-5 cm (vert)
- Ortho heights accuracy: unknown



Data preparation

- Mean tide to tide-free conversion for orthometric heights using:

$$H_{TF} - H_M = (29.6 \sin^2 \varphi - 9.9) \gamma \text{ [cm]}$$

- Computation of g at each benchmark using:

$$g_{BM} = \gamma_{BM} - \frac{\partial T}{\partial r}$$



computed from EGM08

- Computation of W_i from EGM08

*All computations were carried out in a tide-free system
Computations for EGM08 were made using Pavlis&Holmes software*



Un-weighted LS estimate

$$\hat{W}_0^{\text{LVD}} = 62636859.37 \pm \text{?.??} \text{ m}^2/\text{s}^2$$

Weighted LS estimate ($p_i = 1 / H_i^{\text{helm}}$)

$$\hat{W}_0^{\text{LVD}} = 62636860.16 \pm \text{?.??} \text{ m}^2/\text{s}^2$$

Difference: $\delta\hat{W}_0^{\text{LVD}} = 0.79 \text{ m}^2/\text{s}^2 \approx 8 \text{ cm} !$

() estimated zero-height level of the Hellenic mainland LVD as given by Sima et al. (EUREF 2009):*

$$\hat{W}_0^{\text{LVD}} = 62636859.44 \text{ m}^2/\text{s}^2$$



| Height threshold for used BMs | \hat{W}_0^{LVD} | | Difference |
|-------------------------------|-------------------|--------------------------------------|------------|
| | Un-weighted | Weighted ($p_i = 1/H_i^{helm}$) | |
| < 200 m, 514 pts | 62636860.04 | 62636860.20 | ~ 1.6 cm |
| < 500 m, 866 pts | 59.90 | 60.19 | ~ 3.0 cm |
| < 1000 m, 1308 pts | 59.65 | 60.17 | ~ 5.3 cm |
| < 1500 m, 1487 pts | 59.45 | 60.17 | ~ 7.3 cm |
| < 2000 m, 1535 pts | 59.37 | 60.16 | ~ 8.1 cm |



Evidence of a height-correlated bias in the data

| | \hat{W}_0^{LVD} | | Difference |
|--------------------|-------------------|--|------------|
| | Un-weighted | Weighted ($p_i = 1 / H_i^{helm}$) | |
| < 200 m, 514 pts | 62636860.04 | 62636860.20 | ~ 1.6 cm |
| < 500 m, 866 pts | 59.90 | 60.19 | ~ 3.0 cm |
| < 1000 m, 1308 pts | 59.65 | 60.17 | ~ 5.3 cm |
| < 1500 m, 1487 pts | 59.45 | 60.17 | ~ 7.3 cm |
| < 2000 m, 1535 pts | 59.37 | 60.16 | ~ 8.1 cm |



Evidence of a height-correlated bias in the data

More robust estimates due to data weighting

$$\hat{W}_O^{LVD}$$

| | Un-weighted | Weighted ($p_i = 1 / H_i^{\text{helm}}$) | Difference |
|--------------------|-------------|---|------------|
| < 200 m, 514 pts | 62636860.04 | 62636860.20 | ~ 1.6 cm |
| < 500 m, 866 pts | 59.90 | 60.19 | ~ 3.0 cm |
| < 1000 m, 1308 pts | 59.65 | 60.17 | ~ 5.3 cm |
| < 1500 m, 1487 pts | 59.45 | 60.17 | ~ 7.3 cm |
| < 2000 m, 1535 pts | 59.37 | 60.16 | ~ 8.1 cm |



NUMERICAL RESULTS

Evidence of a height-correlated bias in the data

Even more robust estimates due to data weighting

$$\hat{W}_0^{LVD}$$

| | Un-weighted | Weighted ($p_i = 1 / (H_i^{helm})^2$) | |
|--------------------|-------------|--|----------|
| < 200 m, 514 pts | 62636860.04 | 62636860.12 | ~ 0.8 cm |
| < 500 m, 866 pts | 59.90 | 60.12 | ~ 2.2 cm |
| < 1000 m, 1308 pts | 59.65 | 60.12 | ~ 4.8 cm |
| < 1500 m, 1487 pts | 59.45 | 60.12 | ~ 6.8 cm |
| < 2000 m, 1535 pts | 59.37 | 60.12 | ~ 7.6 cm |

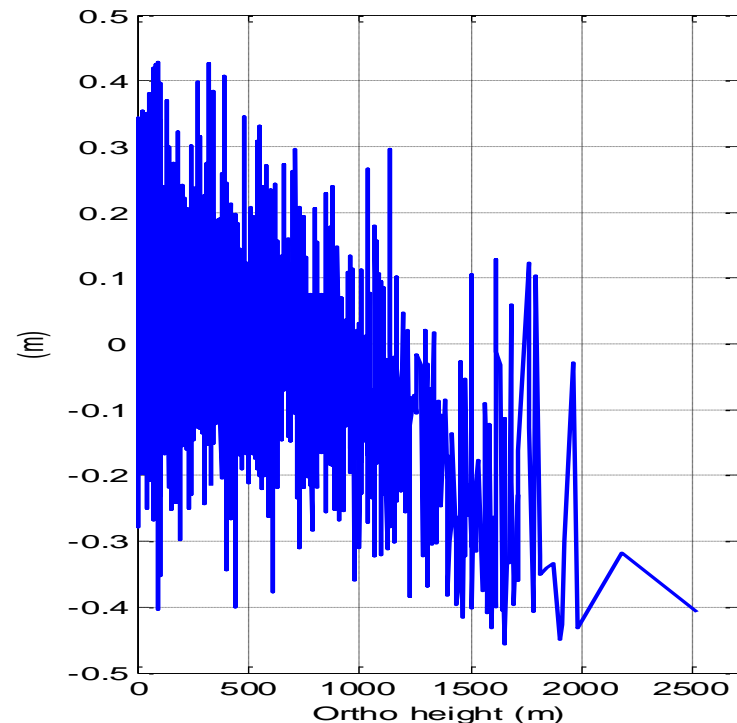
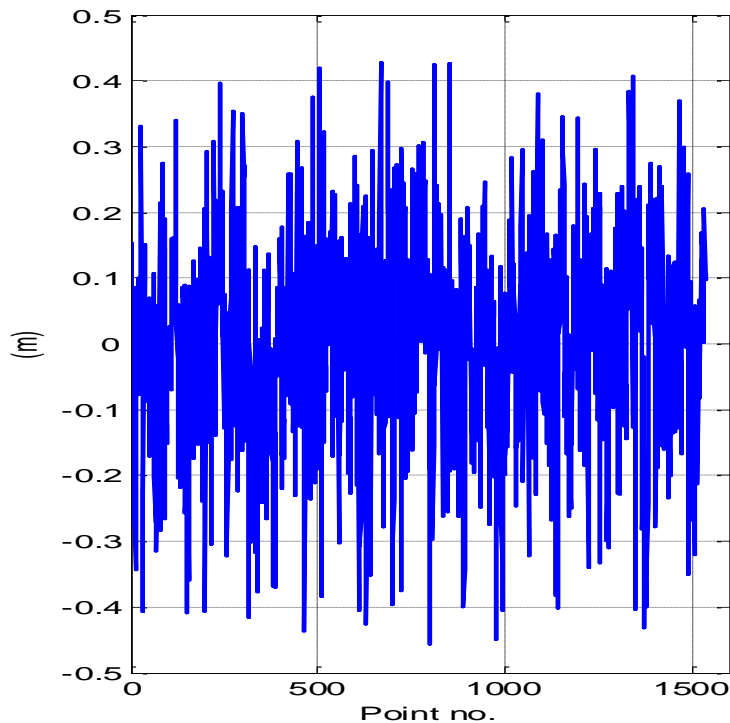


W_o-reduced Helmert ortho heights

(adjusted residuals from unweighted LSA)

$$e_i = H_i^{\text{helm}} - \frac{\hat{W}_o^{\text{LVD}} - W_i}{g_i^{\text{helm}}}$$

$$\hat{W}_o^{\text{LVD}} = 62636859.37 \text{ m}^2/\text{s}^2$$



max = 0.429 m min = -0.456 m **mean = 0.000 m** σ = 0.150 m



Revised model

(taking into account height-correlated data errors)

$$H_i^{\text{helm}} = \underbrace{\frac{W_o^{\text{LVD}} - W_i}{g_i + 0.0424 H_i^{\text{helm}}}} + \lambda H_i^{\text{helm}}$$

LSA results

| | \hat{W}_o^{LVD} | $\hat{\lambda}$ |
|--|--------------------------|-------------------------|
| Un-weighted | 62636860.30 | -1.882×10^{-4} |
| Weighted $p_i = (1/H_i^{\text{helm}})^{1/2}$ | 62636860.28 | -1.832×10^{-4} |
| Weighted $p_i = (1/H_i^{\text{helm}})$ | 62636860.23 | -1.725×10^{-4} |
| Weighted $p_i = (1/H_i^{\text{helm}})^2$ | 62636860.12 | 1.339×10^{-4} |
| Weighted $p_i = (1/H_i^{\text{helm}})^3$ | 62636860.10 | 80.201×10^{-4} |



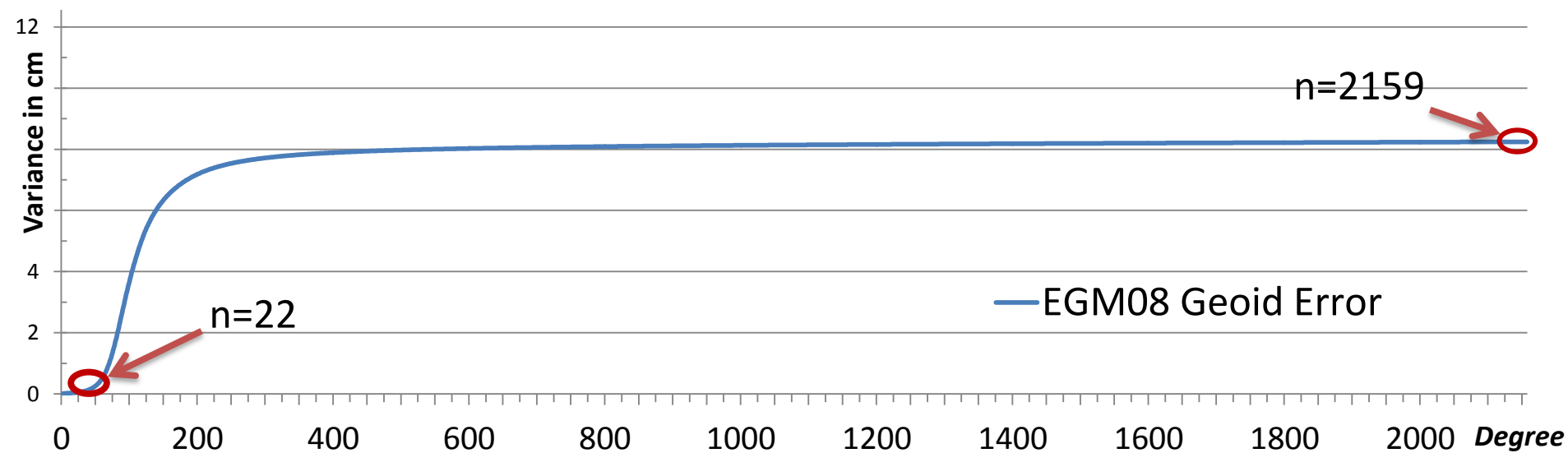
W_o-reduced Helmert ortho heights (adjusted residuals)

$$e_i = H_i^{\text{helm}} - \frac{\hat{W}_o^{\text{LVD}} - W_i}{g_i + 0.0424 H_i^{\text{helm}}} - \hat{\lambda} H_i^{\text{helm}}$$

| | Un-weighted | Weighted $p_i = (1 / H_i^{\text{helm}})^{1/2}$ | Weighted $p_i = (1 / H_i^{\text{helm}})$ | Weighted $p_i = (1 / H_i^{\text{helm}})^2$ | Weighted $p_i = (1 / H_i^{\text{helm}})^3$ |
|------|--------------------|--|--|--|--|
| Max | 0.415 | 0.412 | 0.405 | 0.340 | 0.235 |
| Min | -0.481 | -0.479 | -0.474 | -0.821 | -20.684 |
| Mean | 0.000 | 0.000 | 0.000 | -0.144 | -4.140 |
| Std | 0.125 | 0.125 | 0.126 | 0.188 | 3.589 |



EGM08 'bias' estimate over the Hellenic LVD



Cumulative error d/o 22 (~880km) : 4.83 cm

Cumulative error d/o 2159 (~ 10km) : 8.22 cm

> 'Bias' attributed to EGM08 : 3.39cm <



- ✓ A new estimate for the W_0 for continental Greece was computed based on Helmert orthometric heights and a high resolution geopotential model
- ✓ A LS adjustment using proper weighting was necessary in order to overcome the height correlation bias detected
- ✓ The results are also affected by a 'bias' introduced by the used geopotential model
- Future work involves the estimation of W_0 using other techniques and data including geoid heights and GOCE based geopotential models





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