A New Method Computing Vertical Deflections From Satellite Altimeter Data

L.F. Bao, Yang Lu
(Laboratory of Dynamical Geodesy Institute of Geodesy and Geophysical,
Chinese Academy of Sciences, Wuhan 430077, China)

Abstract For enhancing resolution of vertical deflections and gravity anomaly from altimetry data, a new computing vertical deflections method was brought forward. Firstly, remove influence of sea surface topography from mean sea surface height of altimetry data, and regard results after removing as altimetry geoid. Transform data points to a new Cartesian coordinates composing of osculating tangent plane and normal of reference ellipsoid. Then, simulate tangent plane of regional geoid by using a least squares approach with least total distance squares between discrete points and simulative plane in order to determinate mean value of each component of vertical deflections. We make an imitated experimentation in South China Sea. Comparisons between our 5′ ×5′ grid vertical deflections from introduced method and values from other methods, the precision of η is 0.89″ and precision of ξ is 0.84″ compared with values from model, it is correspond to other methods.

Key words: Altimetry, Vertical deflections, osculating tangent plane, least squares approach
Subject classification number: P738

Introduction

Along with development of altimetry technology, there are many altimetry satellite data were used in associative work out high resolution of oceanic gravity anomaly. Such as Sandwell D.T. (1997), and Hwang CW(1998) [1~2] computing 2′ ×2′ global gravity anomaly by combination of Geosat/GM, ERS1 altimetry data. Haiying Wang[3] have determined China South Sea 2′ ×2′ global gravity anomaly by combination of Geosat/GM, ERS1 and T/P altimetry data. Li Jian-cheng[4] has got 2.5′ ×2.5′ gravity anomaly in China sea and vicinity by combination of Geosat/GM, ERS2 and T/P altimetry data. They all adopt the method computing gravity anomaly from vertical deflections in crossover of satellite tracks, which is so-called vertical deflections method. According to comparisons, this approach is the best method computing high resolution gravity anomaly at present[5]. Nevertheless, for values of vertical deflections are localized at crossover of satellite tracks and density of crossovers is limited after all, it is difficult ascertaining higher resolutions gravity anomaly. It is very important to ameliorate arithmetic of computing vertical defelctions. At the present time, the main method in computing vertical deflections are method computing first difference of mean sea surface height (or geoid height) [6-7], method computing time differential coefficient along satellite tracks, and method computing vector product of ascend arc and descend arc [1-4][8-9]. The former employ data on the grid into compute, the precondition there has grid mean sea surface height (or geoid height) is difficult for actual altimetry data. Both of latter need crossovers of tracks.

In this issue, we bring forward “geoid quasi tangent plane method” computing vertical deflections taking full advantage of altimetry data in order to increasing resolution of vertical deflection, and ascertaining high-resolution gravity anomaly.

Theory and method

First, grids are made along longitudes and latitudes. In each grid, then, a curvilinear coordinates is compose of longitude, latitude and outer normal of reference ellipsoid with left-bottom point of grid as origin. And take arc length from origin to data point along latitude as u_1 axis, arc length from origin to data point along longitude as u_2 axis, and mean sea surface height as u_3 axis.

The original data are saved in the form of geodetic coordinates. For the convenience of computing, it is a good idea to transform data pasting pretreatment to Cartesian coordinates, as figure 1 show. Make a tangent plane of reference ellipsoid with left-bottom of grid being osculating point. In this tangent plane, take projection of longitude circle as X-axis, projection of latitude circle as Y-axis, and outer normal of reference ellipsoid as Z-axis.

In this Cartesian coordinates, X axis is the projection of u_1 axis of the original curvilinear coordinates along outer normal, Y axis is the projection of u_2 axis of the original curvilinear coordinates along outer normal, and Z axis hold the line. In Cartesian coordinates, the common equation of a plane is $Ax+By+Cz+D=0$, where *A*, *B*, *C*, *D* are constants. Generally speaking, *D* unequal to zero and we can predigest simulative planar equation as $Ax+By+Cz+1=0$.

According to knowledge of analytic geometry, the vertical distance between scattered point and simulative plane is,

$$d_n = \frac{(Ax+By+Cz+1)}{\sqrt{A^2+B^2+C^2}} \quad .$$

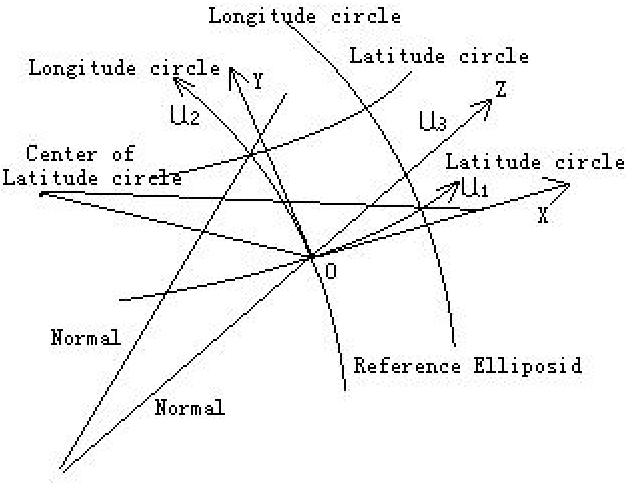

Figure 1:Transform of Curvilinear coordinates and Grid coordinates

And there is $$\sum_{n=1}^{N} d_n^2 = \sum_{n=1}^{N} \frac{(Ax+By+Cz+1)^2}{A^2+B^2+C^2},$$

where *N* is the total number of discrete points. We can express it as $f(A,B,C)=\sum_{n=1}^{N} d_n^2$.

For the convenience of expressing, following, we substitute $\sum$ for $\sum_{n=1}^{N}$.

It is supposed that, there is only one simulative plane meeting conditions when N is big enough, according with necessary conditions by an algorithm using a least squares approach. So there are $f'_A(A_0, B_0, C_0) = 0,\ f'_B(A_0, B_0, C_0) = 0,\ f'_C(A_0, B_0, C_0) = 0$, and

$$\begin{cases} (A^2 + B^2 + C^2)\sum(Ax_n + By_n + Cz_n + 1)\cdot x_n = A\cdot\sum(Ax_n + By_n + Cz_n + 1)^2 \\ (A^2 + B^2 + C^2)\sum(Ax_n + By_n + Cz_n + 1)\cdot y_n = B\cdot\sum(Ax_n + By_n + Cz_n + 1)^2 \\ (A^2 + B^2 + C^2)\sum(Ax_n + By_n + Cz_n + 1)\cdot z_n = C\cdot\sum(Ax_n + By_n + Cz_n + 1)^2 \end{cases} \tag{1}$$

Predigesting this equation group, we can get three linear correlative equations.

$$\begin{cases} B\sum(Ax_n + By_n + Cz_n + 1)\cdot x_n = A\sum(Ax_n + By_n + Cz_n + 1)\cdot y_n \\ C\sum(Ax_n + By_n + Cz_n + 1)\cdot x_n = A\sum(Ax_n + By_n + Cz_n + 1)\cdot z_n \\ C\sum(Ax_n + By_n + Cz_n + 1)\cdot y_n = B\sum(Ax_n + By_n + Cz_n + 1)\cdot z_n \end{cases} \tag{2}$$

This is an equation group containing three variants with maximal exponent being two. Additional restrictive condition is needed in order to work out results. The tangent plane of geoid can't be vertical with reference ellipsoid, as a rule, there is $C \neq 0$.

Otherwise, for any plane（$D \neq 0$）,when we express this plane with equation $Ax + By + Cz + D = 0$, there is a necessary condition $f'_D(A, B, C, D) = 0$ for computing extremum of $\sum d_n^2$. It means $\dfrac{\partial\left(\sum \dfrac{(Ax + by + Cz + D)^2}{A^2 + B^2 + C^2}\right)}{\partial D} = 0$, i.e. $\sum(Ax + By + Cz + D) = 0$. So there are formulas $A\sum x_n + B\sum y_n + C\sum z_n + N = 0$ and $A\frac{\sum x_n}{N} + B\frac{\sum y_n}{N} + C\frac{\sum z_n}{N} + 1 = 0$. These formulas show that the simulative plane must past the geometric center of discrete points $(\frac{\sum x_n}{N}, \frac{\sum y_n}{N}, \frac{\sum z_n}{N})$.

For the convenience of computing, we shift coordinates from the old coordinates to a new coordinates whose origin is $(\frac{\sum x_n}{N}, \frac{\sum y_n}{N}, \frac{\sum z_n}{N} - 1)$ in primary coordinates. In new coordinates, discrete points$'$ coordinate are expressed as (x', y', z'), it is shown in figure 2.

As forgoing statement, we have already had a restrictive condition, which means that simulative plane will past geometric center of discrete points. It is same for new coordinates, i.e. the simulative plane will also past the new coordinate of geometric center (0, 0, 1). In the new coordinates, the simulative

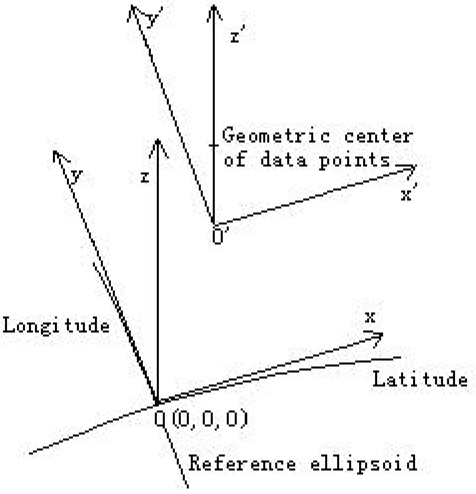

Figure 2:Transform of coordinates

planar equation will become $A'x_n' + B'x_n' + C'z_n' + 1 = 0$. It is the same simulative plane for the new and old coordinates, so we can use (A', B', C') express planar inner normal direction. The transforming equation of coordinates is shown as following:

$$\begin{cases} x_n' = x_n - \frac{\sum x_n}{N} \\ y_n' = y_n - \frac{\sum y_n}{N} \\ z_n' = z_n - \frac{\sum z_n}{N} + 1 \end{cases}$$

And there are relationships:

$\sum x_n' = 0 \qquad \sum y_n' = 0 \qquad \sum z_n' = N.$

In Cartesian coordinates $o' - x'y'z'$, the equation group (2) is still right. Substitute expressions of x', y', z' into equation group, and outspread coefficients of equations:

$$\begin{cases} \sum(y_n'z_n')\cdot A' - \sum(x_n'z_n')\cdot B' - \sum(x_n'y_n')\cdot A'^2 + \sum(x_n'y_n')\cdot B'^2 + \sum(x_n'^2 - y_n'^2)\cdot A'B' = 0 \\ [-N - \sum(x_n'^2 - z_n'^2)]\cdot A' - \sum(x_n'y_n')\cdot B' - \sum(x_n'z_n')\cdot A'^2 - \sum(y_n'z_n')\cdot A'B' + \sum x_n'z_n' = 0 \\ -\sum(x_n'y_n')\cdot A' + [-N - \sum(y_n'^2 - z_n'^2)]\cdot B' - \sum(y_n'z_n')\cdot B'^2 - \sum(x_n'z_n')\cdot A'B' + \sum y_n'z_n' = 0 \end{cases}$$

This is a linear-correlative equation group containing dual-variants with maximal exponent other than one. When we subtract these three equations each other, remove $A'B'$ and B'^2 terms, we can get an equation as following, which just contains B', A' and A'^2 terms.

$$\begin{aligned} &[(\sum x_n'y_n')^2\cdot\sum(x_n'z_n') - \sum(x_n'y_n')\cdot\sum(y_n'z_n')\cdot(N + \sum(x_n'^2 - z_n'^2)) - (\sum y_n'z_n')^2\cdot\sum(x_n'z_n')]\cdot B' \\ &\quad + [(\sum x_n'z_n')^2\cdot\sum(x_n'y_n') - (\sum y_n'z_n')^2\cdot\sum(x_n'y_n') - \sum(x_n'y_n')\cdot\sum(y_n'z_n')\cdot\sum(x_n'^2 - y_n'^2)]\cdot A'^2 \\ &\quad + [\sum(x_n'y_n')\cdot\sum(x_n'z_n')\cdot(N + \sum(x_n'^2 - z_n'^2)) - (\sum x_n'y_n')^2\cdot\sum(y_n'z_n') - (\sum y_n'z_n')^3 \\ &\qquad\qquad - \sum(y_n'z_n')\cdot\sum(x_n'^2 - y_n'^2)\cdot(N + \sum(x_n'^2 - z_n'^2)]\cdot A' \\ &\quad + [\sum(x_n'y_n')\cdot(\sum y_n'z_n')^2 - \sum(x_n'y_n')\cdot(\sum x_n'z_n')^2 + \sum(x_n'z_n')\cdot(\sum y_n'z_n')\cdot\sum(x_n'^2 - y_n'^2)] = 0 \end{aligned}$$

If the coefficient of B' term equals zero, above equation will become an equation containing one variant with maximal exponent being two. We can obtain values of A' directly according to known formula, and then lead A' back to the second formula of above equation group, finally values of B' can be obtained. If the coefficient of B' term is unequal to zero, put the dependence relationship between B' and A', A'^2 into the second formula of above equation group, we can get an equation containing one variant- A' with maximal exponent being three. Because any equation having one variant and maximal exponent being cubic can be transformed into format $x^3 + px + q = 0$, we can get potential results of A' according to known Cardan formula, then lead values of A' back to functional relationship between B' and A', A'^2, finally gain values of B'. Considering of cyber-error, the results of A' and B' got from above steps are just approximation. Regarding values of A', B' as initial value of Newton-iteration, we will obtain final values of A' and B' attaining enough precision. If there is only alone A', what we need do is just put it into relation formula between B' and A', A'^2 and obtain value of B'. If there is more than one A', we should lead A' and B' back to $\sum d_n'^2$ and select those values of A' and B' making $\sum d_n'^2$ at the least, which are we need. Thus, we have obtained orientation of inner normal of simulative plane (A', B', -1). For two components of vertical deflections are defined on westward and southward directions, we must transform values of A', B' into these two directions employing following formula.

$$\eta = -206265 * \arctan(A')$$

$$\xi = -206265 * \arctan(\mathrm{B}')$$ Unit: arc-second

Emulational experimentation and comparisons

At first, a regional 3′ ×3′ gravity anomaly of South China Sea is formed, according with Sandwell’s 2′ ×2′ gravity anomaly. Based on this gravity anomaly, we have constituted a 3600 rank regional geopotential model named IGG_SCS00A[9], and have computed 3′ ×3′ geoid height and two components of vertical deflections, which regarded as model value will be compared with our results at following. Then, the 3′ ×3′ altimetry geoid height is interpolated onto 1′ ×1′ separate uniform grids, and the new geoid height is emulational observation data we needed.

Based on introduced method and using emulational data as observation data, a high resolution (5′ ×5′) vertical deflections are computed in South China Sea from 105° -122° E and 0° -25° N. Table 1 denotes the statistics of results from this method. Figure 3 is the shaded relief map of east- west component of vertical deflections, and figure 4 is the shaded relief of south-north component of vertical deflections.

Item	η	ξ
Data Number	43829	43829
Min. （″）	-41.6090	-28.8108
Max. （″）	65.6915	43.9959
Mean （″）	-6.7536	3.9539
Std. dev. （″）	5.0870	4.4732

Tab 1:The statistics of vertical deflections.

For proving our results, we has computed different values of vertical deflections from geoid quasi tangent plane, from first difference of geoid height with adjacent grid data used, from IGG_SCS00A model, and from Watts vector product method with imitating two lines along grid lines as ascend and descend arc. Hereon, we eliminate those data whose absolute differences form others values exceed 3″ . Tab 2 is the statistics of comparisons.

	Model-Result		Model-difference		Model-Vector		Vector-Result		Difference-Result	
	η	ξ	η	ξ	η	ξ	η	ξ	η	ξ
>±3″	3.02%	2.08%	2.74%	2.98%	2.77%	1.81%	0.11%	0.17%	0.38%	0.91%
Num.	41960	42369	38460	38569	42076	42427	42893	42859	43403	43404
Max	2.9978	2.9969	2.9993	2.9968	-2.9944	-2.9968	2.9802	2.9866	2.9952	2.9980
Min	-2.9998	-2.9996	-2.9977	-2.9973	2.9990	2.9874	-2.9959	-2.9669	-2.9998	-2.9951
Mean	-0.0205	-0.034	-0.0582	0.2775	-0.0269	0.0609	0.0014	-0.1143	0.0358	-0.3400
Std dev	0.8913	0.8410	0.8549	0.8994	0.8633	0.8226	0.5054	0.5190	0.6014	0.6725
Rms	0.8916	0.8417	0.8569	0.9412	0.8637	0.8248	0.5054	0.5314	0.6025	0.7536

Tab2: The statistics of results from comparising difference method.

If regarding values from geopotential model as standard values, following items is obvious from the statistics of results:

1. In the whole research area (0° -25° N, 105° -122° E), precision of η of vertical deflections computed with geoid quasi tangent plane method is 0.89″ , and precision of ξ is 0.84″ . Figure 3 and Figure 4 are their distributing contour map.
2. In this region, both of precision of η and ξ from geoid quasi tangent plane method is as much as from first difference of mean sea surface height (or geoid height)method, and from Watts vector product method.

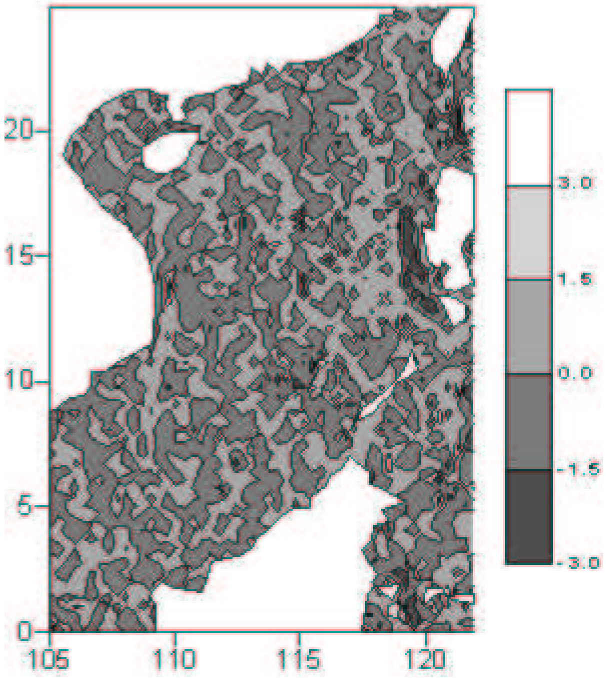

Fig 3: The distributing contour map of difference of η between vertical deflections from model and from geoid quasi tangent plane method

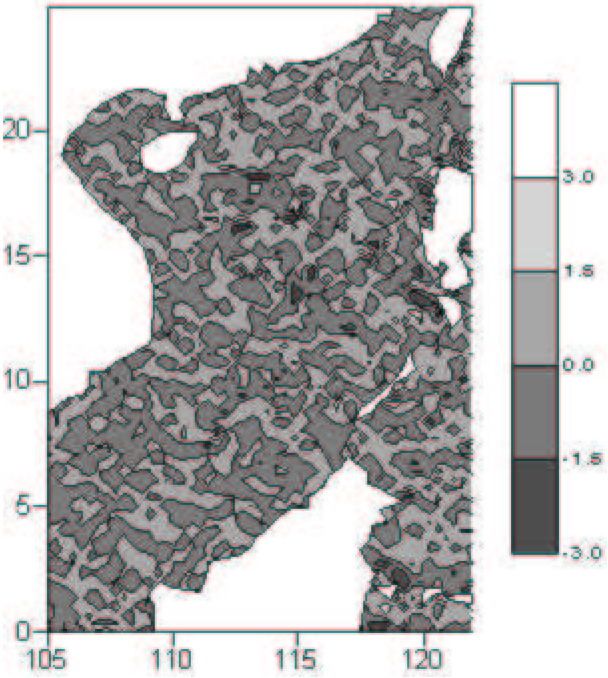

Fig 4: The distributing contour map of difference of ξ between vertical deflections from model and from geoid quasi tangent plane method

Conclusions

During the past decades of years, there are plenty of altimeter data, which provide abundant and valuable data for geophysical, geodesic and oceanography applications. Because the knowledge of a high-accurate and detailed vertical deflections field over the ocean is a basic tool for different applications, it has attracted more and more attention. And the grid value of vertical deflections determined by introduced geoid quasi tangent plane method, as a whole, reflect correctly distribution of vertical deflections. Comparisons with other methods show its possibility and accurate for computing vertical deflections. Without being localized by crossover of satellite tracks, this method can use all data in the grid in order to ascertaining vertical deflections, and there is further information in the result. As a result, it can be used to compute higher resolution vertical deflections, and higher resolution gravity anomaly.

Acknowledge

This work was supported by the National Natural Science Foundation of China (Grant No.49634140) and Key Project of Chinese Academy of Sciences (KZCX2-106).

References

1 Sandwell DT, et al. Marine gravity anomaly from Geosat and ERS-1 satellite altimetry. Journal of Geophysical Research, 1997,102:10039-10054

2 Hwang CW, Inverse Vening-Meinesz formula and deflection geoid formula: applications to the predictions of gravity and geoid over the South China Sea. Journal of Geodesy, 1998,72(4): 304-312

3 Wang Haiying, et al. Recovery of gravity anomalies from satellite data in China seas and vicinity --Ⅱ. Method with along-track vertical deflections. High Technology Letters.2000, 11:46-49

4 LI Jian-cheng, et al. Determination of gravity anomalies over the South China Sea by combination of TOPEX/Poseidon, ERS2 and Geosat altimeter data. Acta Geodaetica et Cartographica Sinica ,2001, 30(3):197-202

5 Huang Mo-tao, et al. On the recovery of gravity anomaly from altimetry data. Acta Geodaetica et Cartographica Sinica , 2001, 30(2):179-184

6 Anzenhofer M., et al. Fully reprocessed ERS-1 altimeter data from 1992 to 1995: reflection of sea level change. Journal of Geophysical Research, 1998,103:8089-8113

7 A.Cazenave, et al. High-resolution mean sea surface computed with altimeter data of ERS-1(geodetic mission) and Topex-Poseidon. Geophys.J.Int, 1996,125:696-704

8 Kim J H. Improved recovery of gravity anomalies from dense altimeter data. In: Rep.437 of Ohio State Univ., 1996

9 A.B. Watts, et al. On the determination of the deflection of the vertical by satellite altimetry. Marine Geodesy, 1984, 8:89-100

10 Yang Lu, High-resolution geoid and seafloor topography character in South China Sea. Progress in Natural Science, 2002 (printing)